HOW TO COMPUTE THE INSTANTANEOUS CENTER OF ROTATION

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Abbreviations

COR: center of rotation

Introduction

In this article a simple geometric method that I used in computing the instantaneous CORs during the downswing will be covered. See Hand-Club Interaction: 2. Mid-Hand Point vs. Center of Rotation for details of how it was used in the argument of hand-club interaction.

An Arbitrary Point on a Line

Figure 1 show an arbitrary point C on a line defined by two end points P_1 and P_2. Since all three points are on the same line, the collinearity condition should hold:

$$\mathbf{x}-\mathbf{r}_2=k\left(\mathbf{r}_1-\mathbf{r}_2\right), \qquad [1]$$

where $\mathbf{x}$, $\mathbf{r}_1$, and $\mathbf{r}_2$ are positions of points C, P_1, and P_2, respectively. k in Eq. 1 is an arbitrary dimensionless ratio. Eq. 1 can be rewritten as

$$\begin{bmatrix} x-x_2 \\ y-y_2 \\ z-z_2 \end{bmatrix} = k\begin{bmatrix} x_1-x_2 \\ y_1-y_2 \\ z_1-z_2 \end{bmatrix}. \qquad [2]$$

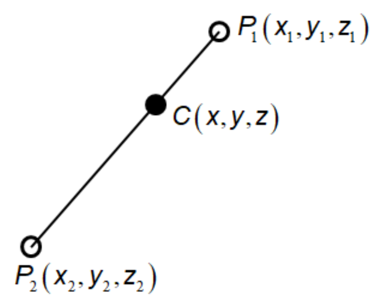

Figure 1. An arbitrary point on a line defined by two end points

Eq. 2 gives

$$x-x_2=\frac{(x_1-x_2)(z-z_2)}{z_1-z_2}, \qquad [3a]$$

$$y-y_2=\frac{(y_1-y_2)(z-z_2)}{z_1-z_2}, \qquad [3b]$$

since

$$\begin{aligned}
x - x_2 &= k\left(x_1 - x_2\right), \\
y - y_2 &= k\left(y_1 - y_2\right), \\
z - z_2 &= k\left(z_1 - z_2\right), \\
k &= \frac{z - z_2}{z_1 - z_2}.
\end{aligned}$$

Eq. 3a and 3b give

$$\left(z_1 - z_2\right)x - \left(x_1 - x_2\right)z = \left(z_1 - z_2\right)x_2 - \left(x_1 - x_2\right)z_2, \qquad [4a]$$

$$\left(z_1 - z_2\right)y - \left(y_1 - y_2\right)z = \left(z_1 - z_2\right)y_2 - \left(y_1 - y_2\right)z_2. \qquad [4b]$$

Eq. 4a and 4b may be combined into single equation:

$$\begin{bmatrix} z_1 - z_2 & 0 & -\left(x_1 - x_2\right) \\ 0 & z_1 - z_2 & -\left(y_1 - y_2\right) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \left(z_1 - z_2\right)x_2 - \left(x_1 - x_2\right)z_2 \\ \left(z_1 - z_2\right)y_2 - \left(y_1 - y_2\right)z_2 \end{bmatrix} \qquad [5]$$

or

$$\begin{bmatrix} \Delta z_{12} & 0 & -\Delta x_{12} \\ 0 & \Delta z_{12} & -\Delta y_{12} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \Delta z_{12} \cdot x_2 - \Delta x_{12} \cdot z_2 \\ \Delta z_{12} \cdot y_2 - \Delta y_{12} \cdot z_2 \end{bmatrix}, \qquad [6]$$

where

$$\begin{bmatrix} \Delta x_{12} \\ \Delta y_{12} \\ \Delta z_{12} \end{bmatrix} = \begin{bmatrix} x_1 - x_2 \\ y_1 - y_2 \\ z_1 - z_2 \end{bmatrix}.$$

Point of Intersection

Figure 2 shows two lines intersecting at common point C . Eq. 5 thus can be expanded for two lines:

$$\begin{bmatrix} \Delta z_{12} & 0 & -\Delta x_{12} \\ 0 & \Delta z_{12} & -\Delta y_{12} \\ \Delta z_{34} & 0 & -\Delta x_{34} \\ 0 & \Delta z_{34} & -\Delta y_{34} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \Delta z_{12} \cdot x_2 - \Delta x_{12} \cdot z_2 \\ \Delta z_{12} \cdot y_2 - \Delta y_{12} \cdot z_2 \\ \Delta z_{34} \cdot x_4 - \Delta x_{34} \cdot z_4 \\ \Delta z_{34} \cdot y_4 - \Delta y_{34} \cdot z_4 \end{bmatrix}. \qquad [7]$$

Eq. 7 is over-determined as it has three unknowns but four equations. Eq. 7 ultimately has the following linear form:

$$\mathbf{Ax} = \mathbf{B}, \qquad [8]$$

where

$$\mathbf{A} = \begin{bmatrix} \Delta z_{12} & 0 & -\Delta x_{12} \\ 0 & \Delta z_{12} & -\Delta y_{12} \\ \Delta z_{34} & 0 & -\Delta x_{34} \\ 0 & \Delta z_{34} & -\Delta y_{34} \end{bmatrix}, \qquad [9a]$$

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \qquad [9b]$$

$$\mathbf{B} = \begin{bmatrix} \Delta z_{12} \cdot x_2 - \Delta x_{12} \cdot z_2 \\ \Delta z_{12} \cdot y_2 - \Delta y_{12} \cdot z_2 \\ \Delta z_{34} \cdot x_4 - \Delta x_{34} \cdot z_4 \\ \Delta z_{34} \cdot y_4 - \Delta y_{34} \cdot z_4 \end{bmatrix}. \qquad [9c]$$

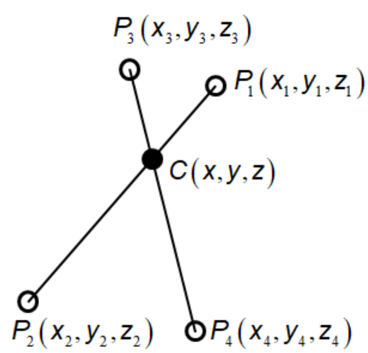

Figure 2. Intersection of two lines

The coordinates of the intersection point ($\mathbf{x}$) can be computed using the least square method:

$$(\mathbf{A'A})^{-1}(\mathbf{A'A})\mathbf{x} = (\mathbf{A'A})^{-1}(\mathbf{A'B}), \qquad [10]$$

where $\mathbf{A'}$ is the transpose matrix of $\mathbf{A}$ and $(\mathbf{A'A})^{-1}$ is the inverse matrix of $\mathbf{A'A}$.

COR of the Club

In the computation of the instantaneous COR of the golf club, the two lines shown in Figure 2 are the positions of the club in two consecutive time points. Point 1 and 3 could be two consecutive positions of the clubhead point while Points 2 and 4 could be those of the grip butt. If the time difference between the adjacent time points is short (e.g. 1/1,000 s), the angle between the lines should be infinitesimal and it is not necessary to worry about the linear motion (translation) of the club. The position of the COR migrates as the swing progresses so the COR is not fixed relative to the club.

Figure 3 shows the trajectory of the instantaneous COR during a drive by a PGA Tour-caliber player. If the two lines shown in Figure 2 cross each other (TB to MD), the COR will remain inside the club. If the lines do not cross (after MD), the COR will lie outside the club.

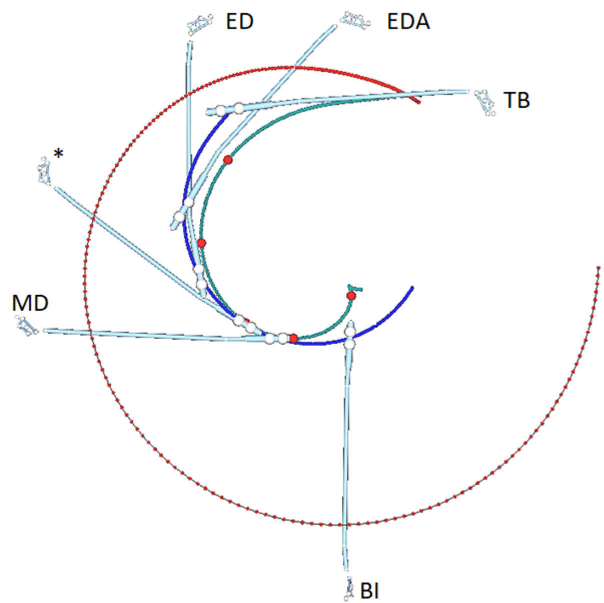

Figure 3. Trajectories of club's instantaneous COR (green line), COM (red line), and MH point (blue line) during downswing. The COR is located close to the clubhead at the beginning of the downswing but moves toward the grip and gets out of the grip near the impact.

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